Assembly Line Grade 7 Determine Ratios Clarification

CCSSM: Grade 7

DOMAIN: Ratios and Proportional Relationships

Cluster: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Standard: 7.RP.1: Compute **unit rates** associated with **ratios** of fractions, including ratios of lengths, areas and other quantities measured in like and or different units.

Standard: **7.RP.2:** Recognize and represent **proportional relationships** between quantities. **2a.** Decide whether two quantities are in a **proportional relationship**.

Clarification

A **RATIO** is a comparison of two numbers or quantities. The ratio of 3 to 5 can be written as 3 out of five, 3:5, or as a fraction where the first number becomes the numerator and the second, the denominator: $\frac{3}{5}$. A **PROPORTION** is a statement that two ratios are equivalent, an equation that illustrates that the two ratios are the same proportional amounts of a whole thing. For example, the proportion $\frac{3}{4} = \frac{18}{24}$ indicates that if a whole thing were divided into 4 equal parts, taking 3 of the 4 parts would be the same proportional amount as taking 18 parts out of the same thing divided into 24 equal parts.

EQUIVALENT RATIOS can be generated using the fact that the number 1 is the identity element for both multiplication and division, that is, any number multiplied or divided by 1 gives the identical original number : $8 \times 1 = 8$, $57 \div 1 = 57$, $\frac{2}{9} \times 1 = \frac{2}{9}$, $\frac{1}{2} \div 1 = \frac{1}{2}$ etc. The number 1, however, can be written in infinitely many different ways, such as $\frac{2}{2}$, $\frac{673}{673}$, $\frac{10{,}000}{10{,}000}$, $\frac{37.5}{37.5}$, $\frac{n}{n}$, $\frac{abc}{abc}$ etc. Therefore, multiplying or dividing a fraction or ratio by a version of 1 keeps its identical proportional value: $\frac{2}{3} \times \frac{7}{7} = \frac{2 \times 7}{3 \times 7} = \frac{14}{21}$. This illustrates that 2 out of 3 is the same proportional part as 14 out of 21.
$\frac{2}{3}$ can also be thought of as $\frac{14}{21}$ reduced to lowest terms by dividing by the common factor, 7. If the numerator and denominator of $\frac{14}{21}$ are divided by 1, written as $\frac{7}{7}$, the result is called the reduced, or simplified form: $\frac{14 \div 7}{21 \div 7} = \frac{2}{3}$ If the numerator and denominator of the reduced form have no common factors, the ratio is said to be in lowest terms. Since there are an infinite

number of fractions that are equivalent to a given ratio such as $\frac{2}{3}$, when comparing ratios, it is convenient to write the fractions in reduced, or simplified form: $\frac{6}{9}=\frac{2}{3}$, $\frac{8}{12}=\frac{2}{3}$, $\frac{10}{15}=\frac{2}{3}$, $\frac{22}{33}=\frac{2}{3}$, $\frac{17}{51}=\frac{2}{3}$, $\frac{42}{63} = \frac{2}{3}$ $\frac{2{,}000{,}000}{3{,}000{,}000} = \frac{2}{3}$ etc.

Classroom Example 1

Is $\frac{7}{13}$ equal to $\frac{42}{78}$?

Method 1: Using multiplication

Answer:

$\frac{7}{13}$ is equal to $\frac{42}{78}$ if there is a common factor by which 7 and 13 are multiplied to get $\frac{42}{78}$:

$\frac{7\times ?}{13\times ?} = \frac{42}{78}$, $\frac{7\times 6}{13\times 6} = \frac{42}{78}$, therefore $\frac{7}{13}$ does equal $\frac{42}{78}$.

Method 2: Using divisors

Answer:

Starting with $\frac{42}{78}$, we can find divisors, or factors of 42 and 78 that would simplify, or reduce $\frac{42}{78}$ to $\frac{7}{13}$: $\frac{42\div ?}{78\div ?} = \frac{7}{13}$. $\frac{42\div 6}{78\div 6} = \frac{7}{13}$.

Often students will do the simplifying, or reducing to lowest terms in several steps. For example, a student might recognize that 42 and 78 are both even numbers, making them divisible by 2. Dividing 42 and 78 by 2 would yield an equivalent ratio: $\frac{42\div 2}{78\div 2} = \frac{21}{39}$, however if we want the ratio in lowest, or simplest terms, as is desirable for comparison purposes, $\frac{21}{39}$ would have to be reduced again, since 21 and 39 have a common factor, 3 : $\frac{21\div 3}{39\div 3}=\frac{7}{13}$. Although successive divisions will give the same final reduced form, it is more efficient to simplify by dividing by the **GREATEST COMMON FACTOR**, as when dividing by 6 rather than 2 to reduce $\frac{42}{78}$. The fraction will be in lowest or simplified form if the numerator and denominator of the fraction have no common factors, which is to say that they are **RELATIVELY PRIME.**

Method 3: Using **Cross-Products**
Answer:

A useful characteristic of proportions is the **CROSS- PRODUCT PROPERTY,** which states that if $\frac{a}{b} = \frac{c}{d}$, then a × d = b × c. Therefore, if $\frac{7}{13} = \frac{42}{78}$, then 7 × 78 = 13 × 42.

7 × 78 = 546 and 13 × 42 = 546, so yes, $\frac{7}{13}$ does equal $\frac{42}{78}$.

Classroom Example 2
Write five other ratios that are equal to $\frac{4}{9}$.
Answers will vary. Some possible answers would include:

$\frac{8}{18} = \frac{4}{9}$, $\frac{12}{27} = \frac{4}{9}$, $\frac{20}{45} = \frac{4}{9}$, $\frac{24}{54} = \frac{4}{9}$, $\frac{28}{63} = \frac{4}{9}$, $\frac{32}{72} = \frac{4}{9}$,

$\frac{36}{81} = \frac{4}{9}$, $\frac{40}{90} = \frac{4}{9}$, $\frac{44}{99} = \frac{4}{9}$, $\frac{48}{108} = \frac{4}{9}$, $\frac{52}{117} = \frac{4}{9}$, $\frac{72}{162} = \frac{4}{9}$

Students might be encouraged to check their answers by reducing or simplifying their fractions to see if they do, in fact, reduce to $\frac{4}{9}$. This would also offer the opportunity to compare and contrast the efficiency of dividing by the greatest common factor, such as dividing by 10 when reducing $\frac{40}{90}$, compared with successive divisions in cases when the greatest common factor is not obvious, as in $\frac{72}{162}$: $\frac{72 \div 2}{162 \div 2} = \frac{36}{81}$, $\frac{36 \div 9}{81 \div 9} = \frac{4}{9}$

In the Assembly Line puzzle, players must demonstrate their knowledge of ratios and proportion by using gear ratios to label premium★ cans. The left gear controls the placement of cans on the belt, and the right, the labeling action. To achieve the correct proportion of premium★ cans to total cans in the collection bins, gamers must study visual clues. Clues include the "can counter" array of small square lights at the top of the screen that matches the layout of the collection bin, the numerical relationship between the ★ cans and the total number of cans in the destination bin, and the appearance of each division on the conveyor belt. The visual clues on the belt are especially useful when the player is presented with two possible sets of gears that meet the required gear ratio.

For instance, when the collection bin has place-holders for 3 premium ★ cans out of the 15 total spots reserved for cans, the target ratio is 1 to 5. Given the choice of left gears with 4, 5, or 6 gear teeth, and right gears with 20, 25, and 26 teeth, players must choose between two acceptable combinations to get the required 1 to 5 ratio. The markings on the belt will give the clue as to whether 5 and 25 are the correct gears, or 4 and 20. Note also that in all cases, both original gears on the opening screen of the puzzle must be replaced, even if one is being replaced by a gear with the same number of teeth.