**Lesson Plan**  
Shipping Grade 7 Representing Numbers in Other Bases

**Standard:** Standard 6.0  
Knowledge of Number Relationships and Computation/Arithmetic:  
Students will describe, represent, or apply numbers or their relationships or will estimate or compute using mental strategies, paper/pencil or technology.

**VSC Topic A:** Knowledge of Number and Place Value

**Indicator 1:** Apply knowledge of rational numbers and place value

**Objective a:** Read, write, and represent whole numbers

**Clarification:** The clarification is an explanation of the indicator and objective and how these math concepts appear in the puzzle.

**Materials and/or Set Up:**  
- Number Stretch; Number Stretch Answers, Bases Loaded, Bases Loaded Answers, When does 5 + 5 = 14?, When does 5 + 5 = 14? Answers, Interactive Resource 1, Interactive Resource 1 Notes, Interactive Resource 2, Interactive Resource 2 Notes, Interactive Resource 2 Answers, Interactive Resource 3, Interactive Resource 3 Notes, Interactive Resource 3 Answers, Interactive Resource 1 Phase 2, Assessment, Assessment Answers

**Relevant Vocabulary:** decimal, equation, expanded form, expression, digits, Mayan Number System, numeration system, order, position, Roman numerals, unique

**Note to Teacher** – Students should have attempted level 3 of the Shipping puzzle before this lesson is implemented. It is assumed for this lesson that the students understand exponents and are familiar with the concept that a number raised to a power of zero equals one.

In the implementation of this lesson, it is recommended that the Interactive Resources be projected to encourage a rich and active discussion of math strategies and concepts.

**Activities:**

1. After students have played the level 3 puzzle of Shipping, ask them to share their experiences and strategies.

2. Write the following symbols on the chalkboard: $7$  VII  ___  13

   • Ask: What observations can you make about the four numbers written on the board? *(At this point, the students may not know that VII and ___ are numbers.*) Explain to students that all of these symbols represent quantities in various number systems. In fact, all of the symbols represent the same quantity, 7.

   • Remind students that we use a decimal number system, also called a Base 10 system. Because this is our accepted number system, we do not write or say Base 10 each time we write a number. Ask students to look at the symbol, VII. Ask students: Does anyone know what number system is being used here? *(Roman Numeral system)* Explain to students that in the Roman Numeral system, V represents 5 and I represents 1, so 7 is written VII.

   • Ask students to try to identify the number system for ___. *(Mayan Number System is being used. The short horizontal line represents 5 while each dot on top...)*
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represents a 1.)

- Ask students to hypothesize how 13 could represent the number 7. Explain to students that the Roman and Mayan number systems use entirely different symbols to represent their numbers. (Some number systems use the same digits as our decimal system but in a different way. In this case 7 in Base 10 is the same as 13 in Base 4. The decimal system uses ten unique digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The number after 9 is 10, a combination of two unique digits. In the decimal number system, the order and position of the digits determines the value. For example, 43 and 34 are not the same thing.) Ask students: What does the number 43 represent? (4 tens and 3 ones, or 40 + 3) What does 34 represent? (3 tens and 4 ones, or 30 + 4). How are 52 and 502 different? (52 represents 5 tens and 2 ones, or 50 + 2; 502 represents 5 hundreds, 0 tens and 2 ones, or 500 + 2).

3. Using Number Stretch, complete Part A as a class. Remind students that $10^0 = 1$.

4. Have students consider the number 13 in Base 4. (Point out to students that 13 in Base 4 should be read as “one three in base four” instead of “thirteen in base four.”) In groups, ask students to discuss what they think 13 in Base 4 means and how it could be equivalent to 7 in Base 10. (13 in Base 4 means 1 four and 3 ones, or 4 + 3)

5. Write 13 in Base 4 in expanded form as a class. ($1(4) + 3(1) = 1(4^1) + 3(4^0)$. Students may have to be reminded that $4^0 = 1$)

6. Have students turn to the Number Stretch and complete section B with a partner.

7. Discuss questions 1 – 5 as a class on Bases Loaded. Have the students work in small groups to complete 6 – 9.

8. Distribute When Does 5 + 5 = 14? Ask the students to think about the title of the handout and work in small groups to answer the title question. ($5 + 5 = 14$ in Base 6) Work through Section A as a class. Have the students work with a partner to complete section B. Have the students work independently on section C.

11. Using Interactive Resource 1, ask the students what they notice about the symbol pad. (The digits 0, 1, 2, 3, 4 and 5 are on the symbol pad). Ask students: Why are the digits 6, 7, 8 and 9 missing? (The numbers and problems in this puzzle are in Base 6) Ask students what symbol(s) they are able to decode first. Distribute Interactive 1 Notes and explain that the students will be taking notes on this sheet as they determine which game symbols represent each number or operation symbol. Facilitate a discussion about how to determine the values of the symbols.

9. In Phase 2 of the Shipping puzzle, students must determine the correct number in the given base to describe the number of cartons to use. Using Interactive 1 Phase 2, complete the first problem (Screen 1) as a whole group. The second problem (Screens 2-3) should be completed with a partner. The final problem (Screen 4) should be solved.
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individuals.

10. Using Interactive Resource 2, in conjunction with the Interactive 2 Notes, have students work with a partner to solve the puzzle. Ask students how, if at all, this example differed from the last example. (This example is in Base 8 and the picture at the bottom of the screen was a multiplication problem.)

11. Using Interactive Resource 3 in conjunction with the Interactive 3 Notes, have students work individually to solve the puzzle. Encourage students to take careful notes about the steps that they take along the way. Remind the students that there is no set order in which they must solve for the various symbols. Also remind the students to think in the correct base.

Differentiation Suggestions:

- For students having difficulty grasping the idea of alternate number systems, such as Base 5, use 2 cups and several popsicle sticks to model how numbers are represented in different bases. Explain that if you had the number 14 for example, you would represent that with 1 stick in the “tens cup”, or the one on the left, and 4 sticks in the “ones cup”, or the one on the right. Demonstrate other number systems using this method. For example, demonstrate the 13 in Base 5. Place 1 stick in the left cup and 3 sticks of another color in the right cup. The 1 stick on the left represents 1 group of 5 and the 3 sticks in the right cup represent 3 ones. Go through several examples. Some students may find the cups and sticks helpful while solving the Shipping Puzzle.

- Have students play the Base Partner Game to practice addition, subtraction and multiplication using other bases.

- Have students investigate why other systems may have been created or why they were found useful. For example, the Base 5 system was based on the fingers on one hand and the Base 12 system is helpful because it is divisible by so many numbers (1, 2, 3, 4, 6, 12).

Assessment

- Distribute Assessment resource sheet.

Answers:

1. 13

2.a. 42₅

2.b. 22₁₀
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Follow Up:

- Have the students return to the puzzle to apply what they learned in the lesson. Ask: Did the lesson help you to clarify the math in the puzzle? How? What other strategies could you have used to help you solve the puzzle? Check teacher stats in the game to determine the students’ level of understanding.

- Provide the students with the following scenario:

Suzette, the class math whiz, came in rattling off math facts. She was rattling on, “2 + 3 = 5, 7 – 0 = 7, 3 x 2 = 6,” but then something strange happened! Suzette started speaking nonsense, saying, “2 x 7 = 16, 12 – 5 = 5, 3 x 5 = 17, 52 ÷ 6 = 7.” Had Suzette bumped her head?!

Then Alec said “Nope, she’s doing that crazy math on purpose. I heard her say something about a basement, or a baseball or was it a base guitar? I don’t know, base something.”

Aha! Suzette must be using a base different from our usual Base 10 decimal system. What base is Suzette using? Why did the first 3 math problems sound normal? (Suzette is using Base 8. When dealing with single digits up to 7, Base 8 and Base 10 will yield the same results.)

Real World Connection:

- Provide the students with the following scenario:

Computers use “binary code” to represent characters such as letters and numbers. Binary code is really Base 2; the only numbers are 0 and 1. Write the numbers from 0 through 15 using base 2.

Answers:

<table>
<thead>
<tr>
<th>Decimal</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base 2</td>
<td>0</td>
<td>1</td>
<td>10</td>
<td>11</td>
<td>100</td>
<td>101</td>
<td>110</td>
<td>111</td>
</tr>
<tr>
<td>Decimal</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
</tr>
</tbody>
</table>
Part A: Base 10

1. Complete the expansion of the number 4823
   \[4 \times 1000 + 8 \times (____) + \dots + \times (____)\]

2. What do you notice about all of the numbers in parentheses?

3. We can write the expansion of 4,823 another way:
   \[4 \times 1000 + 8 \times 100 + 2 \times 10 + 3 \times 1 = 4 \times 10^3 + 8 \times 10^2 + 2 \times 10^1 + 3 \times 10^0\]

Now, using the notation above, write the following numbers in expanded form:

   48 = 
   15 =
   211 =

Part B: Base 4

1. What are the unique digits used in the Base 4 Numeration System? ___________

2. How do we count in Base 4?

3. Complete the expansion of the 231 in Base 4.
   \[\_ \times 4^2 + 3 \times (\_ ) + \_ \times (\_ )\]

4. Write the following numbers in expanded form:
   \[25_6\]
   \[12_4\]
   \[33_5\]
Part A: Base 10

1. Complete the expansion of the number 4823

\[ 4(1000) + 8(100) + 2(10) + 3(1) \]

2. What do you notice about all of the numbers in parentheses?  
   *Going right to left, each number is 10 times greater than the last.*

3. We can write the expansion of 4,823 another way:
   \[ 4(1000) + 8(100) + 2(10) + 3(1) = \]
   \[ 4(10^3) + 8(10^2) + 2(10^1) + 3(10^0) \]

Now, using the notation above, write the following numbers in expanded form:

- 48 = \[ 4(10^1) + 8(10^0) \]
- 15 = \[ 1(10^1) + 5(10^0) \]
- 211 = \[ 2(10^2) + 1(10^1) + 1(10^0) \]

Part B: Base 4

5. What are the unique digits used in the Base 4 Numeration System?  \( 0, 1, 2, 3 \)

6. How do we count in Base 4?  \( 0, 1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23… \)

7. Complete the expansion of the 231 in Base 4.

\[ 2(4^2) + 3(4^1) + 1(4^0) \]

8. Write the following numbers in expanded form:

- 256 = \[ 2(6^2) + 5(6^0) \]
- 124 = \[ 1(4^1) + 2(4^0) \]
- 333 = \[ 3(5^2) + 3(5^1) \]
1. What number system do we use? ___________________________________

2. Consider the Base 8 number system.
   What digits would be used in the Base 8 Number System?
   ______________________________________________________________

3. What would 32 in Base 8 mean in Base 10? Explain your answer.
   ______________________________________________________________

4. What would 51 in Base 8 mean in Base 10?
   ______________________________________________________________

5. Consider the Base 4 Number System.
   What digits would be used in the Base 4 system?
   ______________________________________________________________

6. List the digits used in Base 5 _____________________________________________

   Complete the following table, rewriting numbers in Base 5 and Base 10.

<table>
<thead>
<tr>
<th>Base 5</th>
<th>Base 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>3_5 = 3(5^0)</td>
<td>3_{10}</td>
</tr>
<tr>
<td>74_5 = 7(5^1) + 4(5^0)</td>
<td>39_{10}</td>
</tr>
<tr>
<td>2_5</td>
<td></td>
</tr>
<tr>
<td>24_5</td>
<td>1_{10}</td>
</tr>
<tr>
<td></td>
<td>9_{10}</td>
</tr>
</tbody>
</table>
7. List the digits used in Base 6

Complete the following table, rewriting numbers in Base 6 and Base 10.

<table>
<thead>
<tr>
<th>Base 6</th>
<th>Base 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>12₆</td>
<td>15₁₀</td>
</tr>
<tr>
<td>34₆</td>
<td>21₁₀</td>
</tr>
</tbody>
</table>

8. List the digits used in Base 7

Complete the following table, rewriting numbers in Base 7 and Base 10.

<table>
<thead>
<tr>
<th>Base 7</th>
<th>Base 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>66₇</td>
<td>17₁₀</td>
</tr>
<tr>
<td>14₇</td>
<td>42₁₀</td>
</tr>
</tbody>
</table>

9. List the digits used in Base 8

Complete the following table, rewriting numbers in Base 8 and Base 10.

<table>
<thead>
<tr>
<th>Base 8</th>
<th>Base 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>10₈</td>
<td>23₁₀</td>
</tr>
<tr>
<td>22₈</td>
<td>50₁₀</td>
</tr>
</tbody>
</table>
1. What number system do we use? *Base 10 System or Decimal System*

2. Consider the Base 8 number system. What digits would be used in the Base 8 Number System?

   \[0, 1, 2, 3, 4, 5, 6, 7\]

3. What would 32 in Base 8 mean in Base 10? Explain your answer.

   \[32_8 = 3(8^1) + 2(8^0) = 26_{10}\]

4. What would 51 in Base 8 mean in Base 10?

   \[51_8 = 5(8^1) + 1(8^0) = 41_{10}\]

5. Consider the Base 4 Number System. What digits would be used in the Base 4 system?

   \[0, 1, 2, 3\]

6. List the digits used in Base 5 \(0, 1, 2, 3, 4\)

   Complete the following table, rewriting numbers in Base 5 and Base 10.

<table>
<thead>
<tr>
<th>Base 5</th>
<th>Base 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td></td>
</tr>
<tr>
<td>(3_5) = 3(5^0)</td>
<td>3_{10}</td>
</tr>
<tr>
<td>(74_5 = 7(5^1) + 4(5^0))</td>
<td>39_{10}</td>
</tr>
<tr>
<td>(2_5) = 2(5^0)</td>
<td>2_{10}</td>
</tr>
<tr>
<td>(24_5 = 2(5^1) + 4(5^0))</td>
<td>14_{10}</td>
</tr>
<tr>
<td>(1_5) = 1(5^0)</td>
<td>1_{10}</td>
</tr>
<tr>
<td>(14_5 = 1(5^1) + 4(5^0))</td>
<td>9_{10}</td>
</tr>
</tbody>
</table>
7. List the digits used in Base 6 \(0, 1, 2, 3, 4, 5\)

Complete the following table, rewriting numbers in Base 6 and Base 10.

<table>
<thead>
<tr>
<th>Base 6</th>
<th>Base 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(12_6 = 1(6^1) + 2(6^0))</td>
<td>(8_{10})</td>
</tr>
<tr>
<td>(34_6 = 3(6^1) + 4(6^0))</td>
<td>(22_{10})</td>
</tr>
<tr>
<td>(23_6 = 2(6^1) + 3(6^0))</td>
<td>(15_{10})</td>
</tr>
<tr>
<td>(33_6 = 3(6^1) + 3(6^0))</td>
<td>(21_{10})</td>
</tr>
</tbody>
</table>

8. List the digits used in Base 7 \(0, 1, 2, 3, 4, 5, 6\)

Complete the following table, rewriting numbers in Base 7 and Base 10.

<table>
<thead>
<tr>
<th>Base 7</th>
<th>Base 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(66_7 = 6(7^1) + 6(7^0))</td>
<td>(48_{10})</td>
</tr>
<tr>
<td>(14_7 = 1(7^1) + 4(7^0))</td>
<td>(11_{10})</td>
</tr>
<tr>
<td>(23_7 = 2(7^1) + 3(7^0))</td>
<td>(17_{10})</td>
</tr>
<tr>
<td>(60_7 = 6(7^1) + 0(7^0))</td>
<td>(42_{10})</td>
</tr>
</tbody>
</table>

9. List the digits used in Base 8 \(0, 1, 2, 3, 4, 5, 6, 7\)

Complete the following table, rewriting numbers in Base 8 and Base 10.

<table>
<thead>
<tr>
<th>Base 8</th>
<th>Base 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10_8 = 1(8^1) + 0(8^0))</td>
<td>(8_{10})</td>
</tr>
<tr>
<td>(22_8 = 2(8^1) + 2(8^0))</td>
<td>(18_{10})</td>
</tr>
<tr>
<td>(27_8 = 2(8^1) + 7(8^0))</td>
<td>(23_{10})</td>
</tr>
<tr>
<td>(62_8 = 6(8^1) + 2(8^0))</td>
<td>(50_{10})</td>
</tr>
</tbody>
</table>
Section A: Complete the following problems in Base 8.

1. 5 + 2 =
2. 4 + 4 =
3. 7 × 4 =
4. 0 × 5 =
5. 16 ÷ 2 =

Section B: Complete the following problems in Base 5.

1. 4 + 3 =
2. 14 + 4 =
3. 14 - 3 =
4. 2 × 4 =
5. 14 ÷ 3 =

Section C: Complete the following problems in Base 7.

1. 6 + 4 =
2. 4 + 2 =
3. 20 - 12 =
4. 11 - 2 =
5. 3 × 4 =
6. 6 × 5 =
7. 13 ÷ 2 =
8. 51 ÷ 6 =
Section A: Complete the following problems in Base 8.

6. 5 + 2 = 7  
7. 4 + 4 = 10  
8. 7 x 4 = 34  
9. 0 x 5 = 0  
10. 16 ÷ 2 = 7

Section B: Complete the following problems in Base 5.

6. 4 + 3 = 12  
7. 14 + 4 = 23  
8. 14 - 3 = 11  
9. 2 x 4 = 13  
10. 14 ÷ 3 = 3

Section C: Complete the following problems in Base 7.

9. 6 + 4 = 13  
10. 4 + 2 = 6  
11. 20 - 12 = 5  
12. 11 - 2 = 6  
13. 3 x 4 = 15  
14. 6 x 5 = 42  
15. 13 ÷ 2 = 5  
16. 51 ÷ 6 = 6
Interactive 1 Notes

Number System: Base __

0 1 2

3 4 5

+ - x ÷ =
(3_{10} + 7_{10} = 10_{10} = 3_{6} + 11_{6} = 14_{6})
(3_6 \times 11_6 = 3_{10} \times 7_{10} = 21_{10} = 33_6)
$3_6 + 12_6 = 3_{10} + 8_{10} = 11_{10} = 15_6$
Interactive Resource 1 Phase 2, Screen 4
$2_6 \times 5_6 = 2_{10} \times 5_{10} = 10_{10} = 14_6$
Interactive 2 Notes

Number System: Base ____

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

| + | - | x | ÷ | = |

Notes:
Interactive 2 Notes-Answers

Number System: Base 8

Notes:

Picture at the bottom shows $2_8 \times 4_8$
Number System: Base 6

0    1    2
3    4    5
+    -    x    ÷    =

Interactive 3 Notes

Notes:
Interactive 3 Notes-Answers

Number System: Base 6

0 1 2
3 4 5

Notes:

Picture at the bottom shows $9_{10} + 9_{10} = 13_6 + 13_6$
The Base Partner Game

Directions:
1. Cut out the digits and operation symbols below.
2. Determine which Number System you will be working with (for example: Base 5)
3. Take the digits used in the Number System you selected, and place them in the envelope. (If you are working with Base 5 for example, you will put the digits
   0, 1, 2, 3, and 4 in the envelope. If you are working with Base 9 you will place
   0, 1, 2, 3, 4, 5, 6, 7, and 8 in the envelope.)
4. Draw two digits from the envelope then create and answer one addition problem, one subtraction problem, and one multiplication problem. Write down the three problems and their answers.
5. Place the digits back in the envelope and repeat step four.

0  1  2  3  4
5  6  7  8
+  -  x
Lesson Plan
Shipping Grade 7 Representing Numbers in Other Bases

Assessment

1. What is $4 + 6$ in Base 7?

2. It’s Base 5 day in math class and our teacher, Ms. Johnson, wants everyone to keep track of the number of steps they take during math class, using Base 5. I walked 12₅ steps to the pencil sharpener, 4₅ steps to the wastebasket, and then 21₅ steps to turn in my work.

   a. What is the number of Base 5 steps that I took in math class?

   b. What number of steps would that be in Base 10?