

Lesson Plan

Vats High School Functions – Function Rules

CCSSM: High School: Functions

Cluster: Build a function that models a relationship between two quantities.

Standard: F.BF.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Materials and/or Set Up: *Cell Phone Choice; Allowance Action; In the Bank; Interactive Resources 1-5, Assessment, Following Up*

Relevant Vocabulary: sequence, term, function rule, coefficient, constant, slope, ordered pair solution

Note to Teacher – This lesson is designed for Algebra students. Before this lesson, students should have mastered levels 1 and 2 of the Vats puzzle, and they should have experience with level 3. Students should have been introduced to two-variable equations. This lesson requires students to write two function rules, set them equal to each other, simplify, and then use number sense to identify pairs of whole numbers (one for each variable) that are possible solutions. The lesson can be used after the previous Vats lesson or it can be used independently. It is also useful to review divisibility rules before this lesson.

In the implementation of this lesson, it is recommended *Interactive Resources 1-5* be projected to encourage a rich and active discussion of math strategies and concepts.

Activities:

1. After students have played level 1 and 2 of Vats, ask them to share their experiences and strategies.
2. Divide the class into small groups.
3. Using *Cell Phone Choice*, give groups time to brainstorm answers.
4. Have students share their answers and demonstrate how they decided which plan to use. Allow groups to share their strategies.
5. After the students have shared their strategies, point out that the cost of text messaging and picture messaging is irrelevant since it is the same in both situations. Then, if no student group demonstrated the technique shared in *Cell Phone Choice-Answers*, discuss the provided solution strategy with them.
6. Using *Allowance Action*, have students use a strategy similar to the one just shared. Have students complete this problem individually and share their solutions with a partner.
7. Using *In the Bank*, have students work with a partner to solve each number and check answers before moving on to the next question.
8. Challenge students by asking after number 8 on *In the Bank*:
 - a. Are there other solutions that are also correct? (Yes. For example $y = 8$, $x = 8$ (550) AND $y = 13$, $x = 12$ (750))
9. Using *Interactive Resource 1*, lead a discussion about strategies that students have

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used to solve level 3 puzzles. (*Remind students that in level 3 of the Vats puzzle, two vats must be lined up at the same time before the green force field disappears.*)

Have students work in pairs to complete **Interactive Resource 1**.

10. Discuss student solutions and strategies for **Interactive Resource 1**. Discuss the procedure outlined in **Interactive Resource 1-Answers**.

11. Using **Interactive Resource 2**, have students work to complete it independently. Ask pairs of students to discuss and check answers for one another. Ask volunteers to explain and justify their responses.

Differentiation Suggestions:

1. Assign students to work in small groups. For students who need more practice, have them work with **Interactive Resource 3**.
 - Ask students to create a sequence that represents the desired outcome for Vat 11 and then a sequence for Vat 4. Assist as necessary. (*See **Interactive Resource 3 Answers** for more information.*)
 - Divide the small group in two subgroups. Have the first subgroup create a function rule for Vat 11 and the second subgroup create a function rule for Vat 4.
 - Assist students with the process of simplifying and finding whole number solutions.
2. Have students work in small groups. For students who have mastered the process, have them work with **Interactive Resource 4**.
 - Give students time to reason through the scenario. (*There are no whole number solutions.*) Ask students to explain why there cannot be a solution to that scenario. ($2 + 4x = 4 + 8y$ simplifies to: $4x = 2 + 8y$ and then to $2x = 1 + 4y$. *Since you are only interested in whole number solutions, there is no solution. The right side of the equation will always be odd and the left side will always be even.*)
3. For enrichment or extension, use **Interactive Resource 5**.

Assessment

- Distribute the **Assessment** resource sheet.

Answers:

1. 5

2a. Apple bags: 4, Tangerine boxes: 6

2b. 46

Follow Up:

- Have students return to the puzzle to apply what they learned in the lesson. Ask: Did the lesson help you to clarify the math in the puzzle? How so? What other strategies could you have used to help you solve the puzzle? Additionally,

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check teacher stats in the game to determine students' level of understanding.

- Using **Following Up**, have students work to solve the problem.
*(Let h = the number of boxes of hats you must buy and
 k = the number of bags of kazoos you must buy*

The total number of hats is: $3h + 2$

The total number of kazoos is: $11k + 7$

In order to have the same number of hats as kazoos, the equation is:

$$3h + 2 = 11k + 7$$

When simplified:

$$3h = 11k + 5$$

One solution is:

$$h = 9, k = 2$$

Substituting:

Number of hats = $3(9) + 2$ or 29 hats

Number of kazoos = $11(2) + 7$ or 29 kazoos

Answers:

You should buy 9 boxes of hats and 2 bags of kazoos.

There should be 29 people at the party.

Real World Connection:

- Provide students with this scenario:

Your family is planning a camping trip during summer vacation. There are two possibilities for the campground at which you will stay.

- Rest-a-While Campground charges a one-time fee of \$25 for water hook-up, plus \$15 per night.
- Sand-n-Sun Campground charges only \$15 for water hook-up, but charges \$20 per night.

You do want to have the water hook-up. After how many nights will the cost of the two campgrounds be the same? What will that cost be?

(The cost will be the same after 2 nights. If n = the number of nights, $25 + 15n$ is the cost at Rest-a-While and $15 + 20n$ is the cost at Sand-n-Sun. These expressions are equal when $n = 2$. The cost will be \$55. When $n = 2$, both expressions have a value of 55.)

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If you have budgeted \$200 for the cost of the campground, at which one would you be able to stay for a longer time? Explain how you determined the answer.

(You would be able to stay at Rest-a-While for a longer time. You could stay for 11 nights for \$190 since $25 + 15(11) = 190$. At Sand-n-Sun, you could only stay for 9 nights and the cost would be \$195 since $15 + 20(9) = 195$.)

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Cell Phone Choice



You have decided to buy a cell phone. You need to choose a plan. You have narrowed the choices down to two plans. Each plan provides you 200 minutes per month at no additional cost. The two plans are described below:

Plan A: \$10 a month, plus \$0.04 per minute

Plan B: \$0.08 per minute with no monthly charge

In each plan, you can also get unlimited text and picture messaging for an additional charge of \$8.95 per month.

Which plan would you choose? Explain and justify your choice.

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Cell Phone Choice- Answers

You have decided to buy a cell phone. You need to choose a plan. You have narrowed the choices down to two plans. Each plan provides you 200 minutes per month at no additional cost. The two plans are described below:

Plan A: \$10 a month, plus \$0.04 per minute

Plan B: \$0.08 per minute with no monthly charge

In each plan, you can also get unlimited text and picture messaging for an additional charge of \$8.95 per month.

Which plan would you choose? Explain and justify your choice.

*Let x represent the number of minutes used in one month.
Then $10 + 0.04x$ represents the cost for one month using Plan A and
 $0.08x$ represents the cost for one month using Plan B.*

$$10 + 0.04x = 0.08x$$

$$10 = 0.04x$$

$$250 = x$$

What this means: At 250 minutes the cost is the same for both plans.

$$10 + 0.04(250) = 0.08(250)$$

Since the slope of plan B is greater than the slope of plan A (the price of Plan B increases faster than the price of plan A), any time over 250 minutes makes Plan B more expensive than Plan A.

Therefore, students should prefer Plan A if they talk for more than 250 minutes a month and Plan B if they talk for less than 250 minutes a month.

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Allowance Action!



Your friend, Tim, has saved \$150 dollars that he earned by mowing five yards.
You have saved \$230 dollars babysitting for your neighbor.

You receive an allowance of \$30 per week and Tim receives \$50 per week.
Assume that neither one of you spends any money and continues to save your allowance each week, how many weeks must pass before you and Tim have the same amount?

Let x represent the number of weeks

$$\text{Tim's Money} = \underline{\hspace{1cm}} + 50x$$

$$\text{Your Money} = 230 + \underline{\hspace{1cm}}$$

For both amounts to be the same:

$$150 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} \qquad \text{Simplify and solve.}$$

$$x = \underline{\hspace{1cm}}$$

Justify that the value you found for x will result in you and Tim having the same amount of money.

After one more week, who will have more money?

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Allowance Action - Answers

Your friend, Tim, has saved \$150 dollars that he earned by mowing five yards. You have saved \$230 dollars babysitting for your neighbor.

You receive an allowance of \$30 per week and Tim receives \$50 per week. Assume that neither one of you spends any money and continues to save your allowance each week, how many weeks must pass before you and Tim have the same amount?

Let x represent the number of weeks

$$\text{Tim's Money} = 150 + 50x$$

$$\text{Your Money} = 230 + 30x$$

For both amounts to be the same:

$$150 + 50x = 230 + 30x$$

Simplify and solve.

$x = 4$, so four weeks must pass for you and Tim to have the same amount of money.

Justify that the value you found for x will result in you and Tim having the same amount of money.

$$150 + 50(4) = 230 + 30(4)$$

$$350 = 350$$

After one more week, who will have more money?

Tim will have more money because Tim receives a greater allowance per week.

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In the Bank



Your friend, Tim, has saved \$150 dollars that he earned by mowing five yards. You have saved \$230 dollars. You do not receive an allowance, but you earn \$40 for each lawn you mow. Tim receives an allowance of \$50 per week. Assume that you and Tim continue to save your money. In order for you and Tim to have the same amount of money, how many lawns must you mow and how many weeks must Tim get his allowance?

1. How is this problem different from the problem *Allowance Action*?

2. For how many answers does this problem ask?

3. Define a variable and write a function rule that represents the amount of money Tim has saved.

The variable, _____ represents _____

Then $150 + \underline{\hspace{1cm}}$ = the amount of money Tim has saved

4. Define another variable and write a function rule that represents the amount of money you have saved.

The variable, _____ represents _____

Then $230 + \underline{\hspace{1cm}}$ = the amount of money you have saved

5. Combine these two expressions to form an equation.

 $150 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

6. Simplify the equation.

7. Find a pair of whole number values for the variables to make the equation true.

8. What is the amount of money that each of you will have when the amount is the same?

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In the Bank - Answers

Your friend, Tim, has saved \$150 dollars that he earned by mowing five yards. You have saved \$230 dollars. You do not receive an allowance, but you earn \$40 for each lawn you mow. Tim receives an allowance of \$50 per week. Assume that you and Tim continue to save your money. In order for you and Tim to have the same amount of money, how many lawns must you mow and how many weeks must Tim get his allowance?

1. How is this problem different from the problem in Resource 2?
In this problem you must determine two different values: the number of weeks Tim gets an allowance and the number of lawns you must mow.
2. For how many answers does this problem ask?
There will be two answers: the number of weeks Tim receives his allowance and the number of lawns you mow.

3. Define a variable and write a function rule that represents the amount of money Tim has saved.

The variable, x represents the number of weeks Tim receives his allowance

Then $150 + 50x$ = the amount of money Tim has saved

4. Define a variable and write a function rule that represents the amount of money you have saved.

The variable, y represents the number of lawns you mow

Then $230 + 40y$ = the amount of money you have saved

5. Combine these two expressions to form an equation.

$$150 + 50x = 230 + 40y$$

6. Simplify the equation.

$$50x = 80 + 40y \text{ or } -80 + 50x = 40y$$

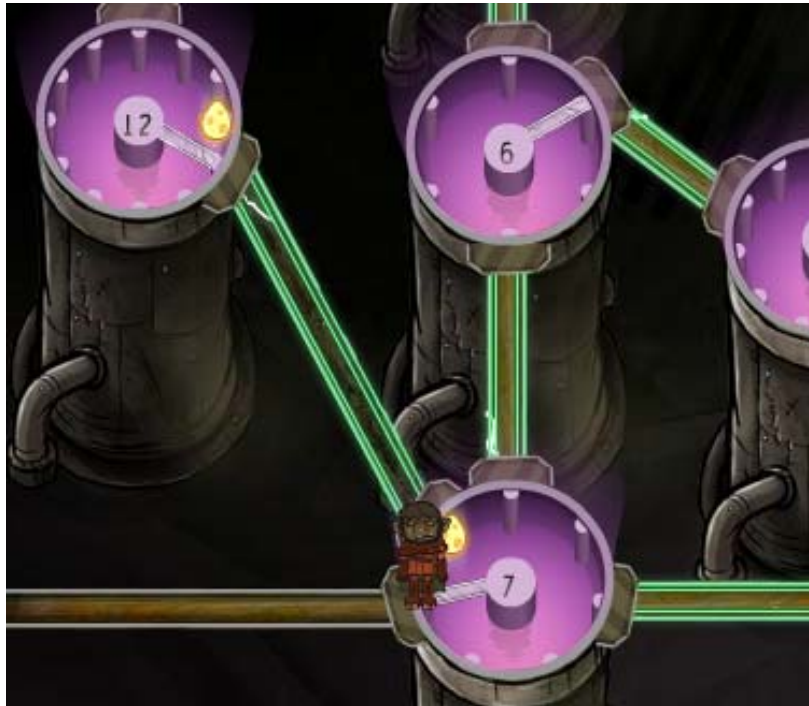
7. Find a pair of whole number values for the variables to make the equation true.

$$x = 4 \text{ and } y = 3$$

8. What is the amount of money that each of you will have when the amount is the same?
You will each have \$350. Emphasize that the values must be substituted in the original expressions to determine this amount.

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Interactive Resource 1



Your goal is to move your avatar out of the 7-vat, across the bridge, and into the 12-vat. Use these steps to help you determine the correct number of times to mix the vats.

1. Write an equation that represents where you want the mixing rod in 7-Vat to stop.

$$m = \underline{\quad} + 7x$$

where m = number of times to mix and x = the number of rotations for the 7-Vat

2. Write an equation represents where you want the mixing rod in 12-Vat to stop.

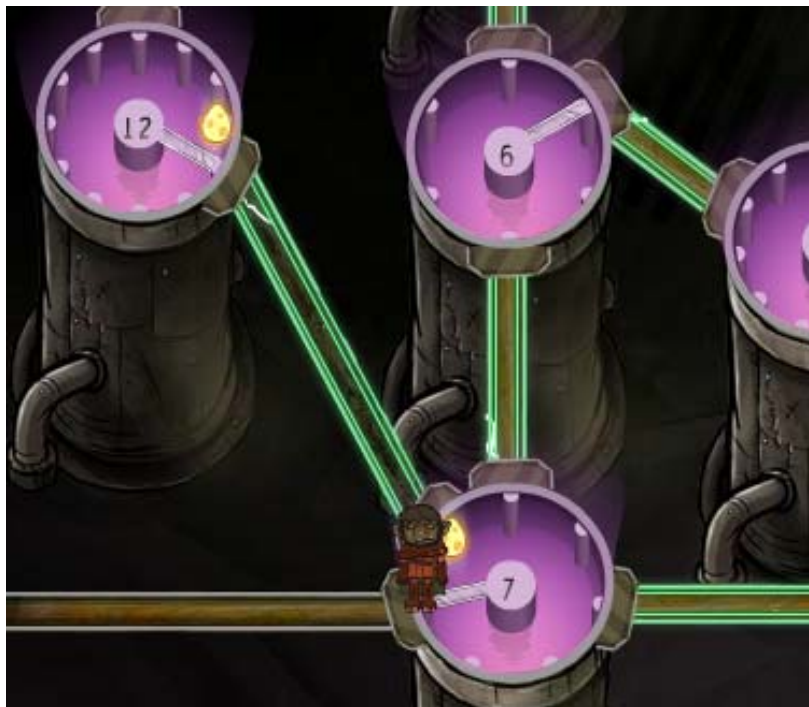
$$m = \underline{\hspace{2cm}}$$

where m = number of times to mix and y = the number of rotations for the 12-Vat

3. Set these equations equal to one another and find a pair of whole number solutions for x and y .
4. Use the values for x and y to determine the value for m .
5. How many times should you mix?

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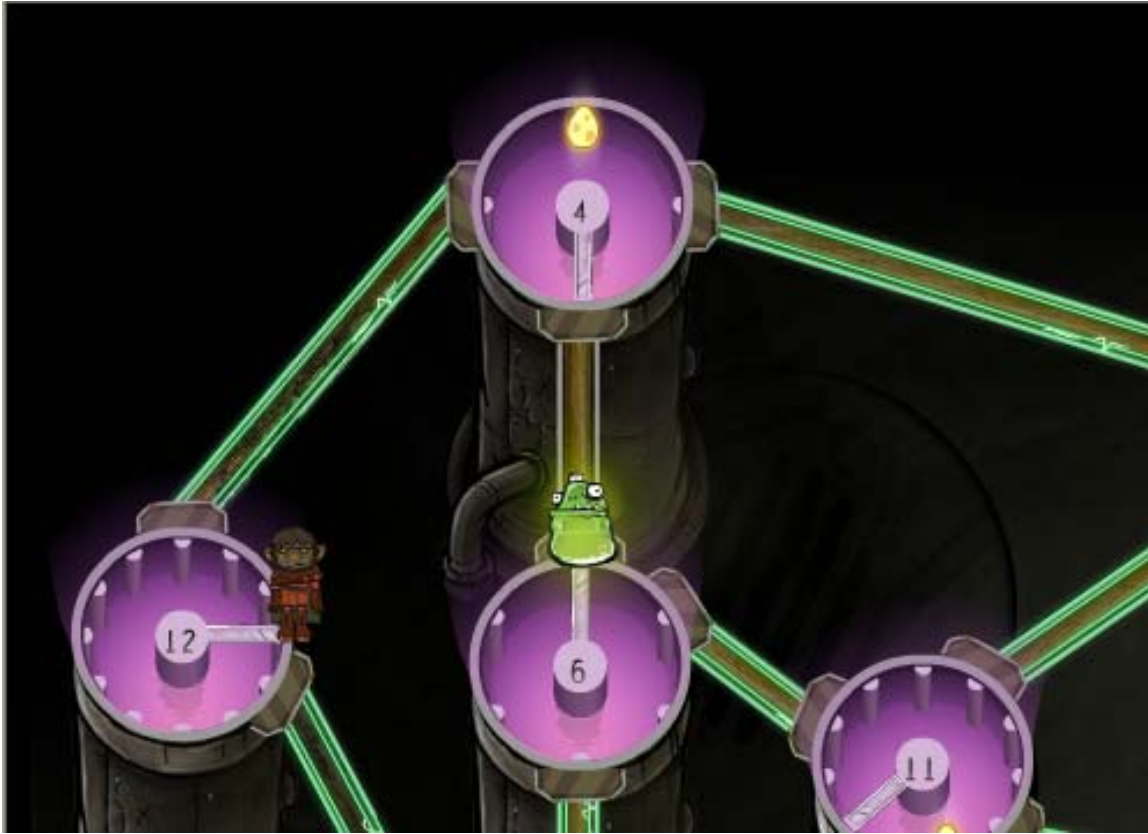
INTERACTIVE RESOURCE 1-Answers



Your goal is to move your avatar out of the 7-vat, across the bridge, and into the 12-vat. Use these steps to help you determine the correct number of times to mix the vats.

1. Write an equation that represents where you want the mixing rod in 7-Vat to stop.
 $m = \underline{1} + 7x$
where m = number of times to mix and x = the number of rotations for the 7-Vat
2. Write an equation represents where you want the mixing rod in 12-Vat to stop.
 $m = \underline{0} + 12y$ or $12y$
where m = number of times to mix and y = the number of rotations for the 12-vat
3. Set these equations equal to one another and find a pair of whole number solutions for x and y .
 $1 + 7x = 12y$ is true when $x = 5$ and $y = 3$. Encourage students find a value for x that results in a multiple of 12.
4. Use the values for x and y to determine the value for m .
 $m = 1 + 7(5)$ and $m = 12(3)$. In both equations the value of m is 36.
5. How many times should you mix?
You should mix 36 times.

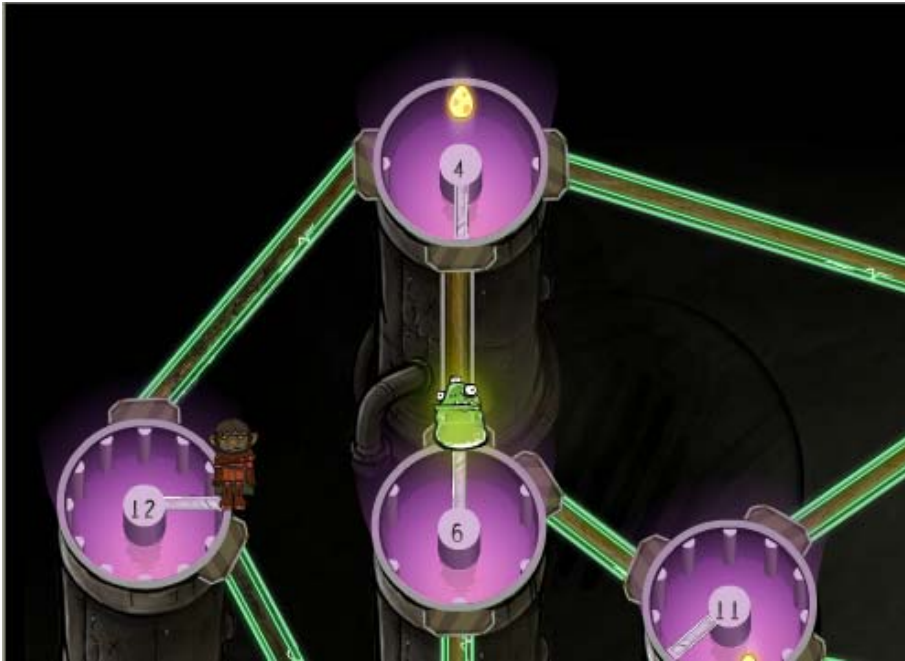
Interactive Resource 2



Imagine that you want to move your avatar from the 12- vat, across the bridge and into the 4-vat. How many times should you mix?

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INTERACTIVE RESOURCE 2-Answers



Using sequences:

For the 12-vat: 9, 21, 33, 45, ...

For the 4-vat: 1, 5, 9, 13, 17, 21, ...

Using equations:

Define variables:

m = the number of times to mix the vat

x = the number of times to mix in the 12-vat

y = the number of times to mix in the 4-vat

Write the rules for the number of times to mix each vat:

$$m = 9 + 12x$$

$$m = 1 + 4y$$

Write equation showing that the number of times to mix must be equal:

$$9 + 12x = 1 + 4y$$

Simplify:

$$8 + 12x = 4y$$

$$2 + 3x = y$$

Find a solution:

$$x = 1 \text{ and } y = 5$$

Answer: You should mix 21 times.

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Interactive Resource 3



Imagine that you want to move your avatar from the 11-vat to the 4-vat. How many times should you ‘mix’?

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INTERACTIVE RESOURCE 3-Answers



Using sequences:

For the 11-vat: 4, 15, 26, 37, 48, ...

For the 4-vat: 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, ...

Using equations:

Define variables:

m = the number of times to mix the vat

x = the number of times to mix in the 11-vat

y = the number of times to mix in the 4-vat

Write the rules for the number of times to mix each vat:

$$m = 4 + 11x$$

$$m = 1 + 4y$$

Write equation showing that the number of times to mix must be equal:

$$4 + 11x = 1 + 4y$$

Simplify:

$$3 + 11x = 4y$$

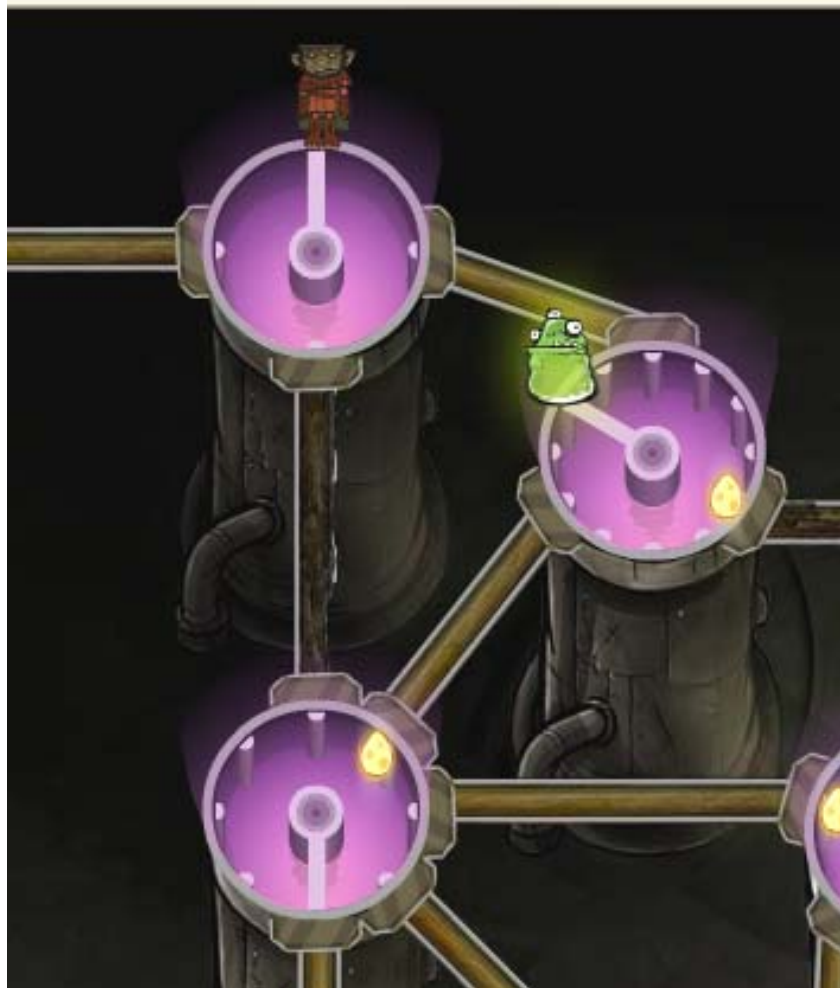
Find a solution:

$$x = 3 \text{ and } y = 9$$

Answer: You should mix 37 times.

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Interactive Resource 4

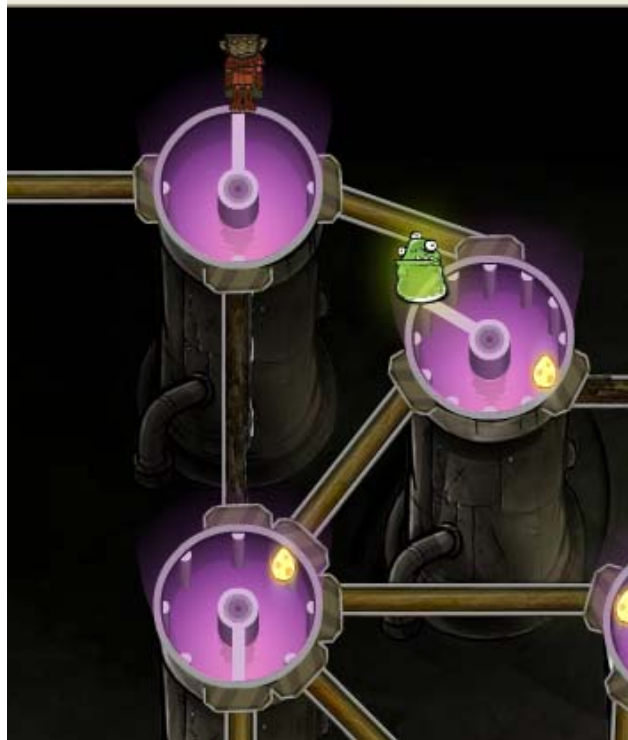


How many times do you have to mix in order to move your Avatar one vat down in *one turn*?

Show your work. Explain and justify your solution.

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Interactive Resource 4 -Answers



How many times do you have to mix in order to move your Avatar one vat down in *one turn*?

It is not possible to move the Avatar one vat down in one move.

Show your work. Explain and justify your solution.

The avatar is located in a 4-vat. He must move either to an 8-vat or to a 12-vat.

For the 8-vat:

$$2 + 4x = 4 + 8y$$

Simplify:

$$4x = 2 + 8y$$

$$2x = 1 + 4y$$

There are no whole number solutions for x and y since the left side of the equation is always an even number while the right side is always an odd number.

For the 12-vat:

$$1 + 4x = 2 + 12y$$

Simplify:

$$4x = 1 + 12y$$

Once again, the left side of the equation is always even while the right side is always odd.

Interactive Resource 5



Find a one-move solution to move your avatar from the 4-vat into the 10-vat.

Show your work. Explain and justify your answer.

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INTERACTIVE RESOURCE 5 -Answers



Find a one-move solution to move your avatar from the 4-vat into the 10-vat.

Show your work. Explain and justify your answer.

$$1 + 4x = 3 + 10y$$

$$4x = 2 + 10y$$

$$x = 3, y = 1$$

You must mix 13 times.

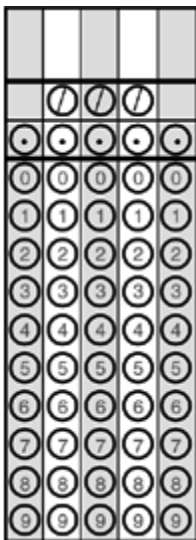
$$1 + 4(3) = 13 \text{ and } 3 + 10(1) = 13.$$

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Assessment

If the value of x is 3 in the equation below, what is the value of y ?

$$7x + 5 = 5y + 1$$



2.

Part A

You want to make a fruit salad that uses an equal number of tangerines and apples. You have 6 apples at home and 4 tangerines. The grocery store sells apples in bags of 10 and tangerines in boxes of 7. It's against your principles to waste any food. How many apple bags and how many tangerine boxes do you need to buy in order not to use the same number of tangerines as apples and still not waste any of the fruit?

Part B

What is the total number of apples and the total number of tangerines that will be in your salad?

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Following Up



You are planning a party. Birthday hats come in boxes of 3 and kazoos come in bags of 11. You have two birthday hats and seven kazoos left over from a previous party. In order to give each person at the party one hat and one kazoo, how many people should attend the party? How many bags of kazoos and how many boxes of hats should you buy? (Do not have any hats or kazoos left over.)

